Abstract.

Given a sequence of numbers $\{p_n\}$ in $[0,1]$, consider the following experiment. First, we flip a fair coin and then, at step n, we turn the coin over to the other side with probability $p_n, n \geq 2$. What can we say about the distribution of the empirical frequency of heads as $n \to \infty$? We show that a number of phase transitions take place as the turning gets slower (i. e. p_n is getting smaller), leading first to the breakdown of the Central Limit Theorem and then to that of the Law of Large Numbers. It turns out that the critical regime is $p_n = \text{const}/n$. Among the scaling limits, we obtain Uniform, Gaussian, Semicircle and Arcsine laws.