Abstract

In the context of functional data analysis, probability density functions as non-negative functions are characterized by specific properties of scale invariance and relative scale which enable to represent them with the unit integral constraint without loss of information. On the other hand, all these properties are a challenge when the densities need to be approximated with spline functions, including construction of the respective B-spline basis. The Bayes space methodology of density functions enables to express them as real functions in the standard L2 space using the clr transformation. The resulting functions possess the zero integral's constraint. This is a key to propose a new B-spline basis, holding the same property, and consequently to build a new class of spline functions, called compositional splines, which can approximate probability density functions in a consistent way. The contribution provides also construction of smoothing by compositional splines and possible orthogonalization of the B-spline basis which might be useful in some applications. Finally, a concise analysis using compositional splines is demonstrated with anthropometric data for the case of functional principal component analysis.