

Abstract: For general covariance matrix, we derive the asymptotic risk of LASSO in the limit of n and p going to infinity with fixed ratio n/p . A phase boundary is precisely established in the phase space. Above this boundary, LASSO perfectly recovers the signals with high probability. Below this boundary, LASSO fails to recover the signals with high probability. While the values of the non-zero elements of the signals do not have any effect on the phase transition curve, our analysis shows that the curve does depend on the signed pattern of the nonzero values of the signal for non-i.i.d. covariance matrix. Underlying our formalism is a recently developed efficient algorithm called approximate message passing (AMP) algorithm. We generalize the state evolution of AMP from i.i.d. case to general case. Extensive computational experiments confirm that our theoretical predictions are consistent with simulation results on moderate size system.