A Test for Multivariate ARCH Effects

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Abstract

This paper extends Engle's *LM* test for ARCH affects to multivariate cases. The size and power properties of this multivariate test for ARCH effects in VAR models are investigated based on asymptotic and bootstrap distributions. Using the asymptotic distribution, deviations of actual size from nominal size do not appear to be very excessive. Nevertheless, there is a tendency for the actual size to overreject the null hypothesis when the nominal size is 1% and underreject the null when the nominal size is 5% or 10%. We find that using a bootstrap distribution for the multivariate *LM* test is generally superior in achieving the appropriate size to using the asymptotic distribution when (1) the nominal size is 5%, (2) the sample size is small (40 observations) and/or the VAR system is stable. With the small sample, the power of the test using the bootstrap distribution appears also better at the 5% nominal size.

Key words: Engle Test, VAR model, Stability, Multivariate ARCH, Bootstrap

JEL classification: C32, C15

Running title: Test for Multivariate ARCH Effects

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This paper extends Engle's *LM* test for ARCH affects to multivariate cases. The size and power properties of this multivariate test for ARCH effects in VAR models are investigated based on asymptotic and bootstrap distributions. Using the asymptotic distribution, deviations of actual size from nominal size do not appear to be very excessive. Nevertheless, there is a tendency for the actual size to overreject the null hypothesis when the nominal size is 1% and underreject the null when the nominal size is 5% or 10%. We find that using a bootstrap distribution for the multivariate *LM* test is generally superior in achieving the appropriate size to using the asymptotic distribution when (1) the nominal size is 5%, (2) the sample size is small (40 observations) and/or the VAR system is stable. With the small sample, the power of the test using the bootstrap distribution appears also better at the 5% nominal size.

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1. Introduction

Since the seminal paper by Engle (1982) autoregressive conditional heteroscedasticity (ARCH) models have been extensively used in applied research. Engle also showed that the ordinary least squares (OLS) estimator is not consistent if ARCH effects are present. He suggested a Lagrange Multiplier (*LM*) test for testing for ARCH effects, which is regularly used as a diagnostic test in regression analyses. This test was introduced for single equation cases. In this paper we extend this test method to test for multivariate ARCH effects in the vector autoregressive (VAR) model. Since the introduction of the VAR model by Sims (1980) this model has been extensively used in applied research. Thus, performing tests for multivariate ARCH effects can be of interest to the practitioners. The aim of the present study is to investigate the size and power properties of a multivariate version of Engle's test for ARCH effects under different situations of stability and instability and of small and moderate sample sizes. The properties of the test will be investigated using asymptotic and bootstrap distributions.

We investigate the size and power properties for 1%, 5% and 10% significance levels. A wide number of parameters will be used in the VAR model to make the results as representative as possible.

This paper is organized as the follows. Section 2 describes the VAR model and the *LM* test for testing ARCH effects in a multivariate perspective. Section 3 introduces our simulation design. Section 4 presents the simulation results. The conclusions are provided in the last section.

2. The VAR Model and Multivariate ARCH Effects

Consider the following vector autoregressive of order p, VAR(p), process:

$$y_t = c + A_1 y_{t-1} + \dots + A_p y_{t-p} + \varepsilon_t , \tag{1}$$

where

y_t = a vector of n variables,

c = a vector of n intercepts,

ε_t = a vector of n error terms, and

A_r = an $n \times n$ matrix of coefficient matrix for lag order r.

Suppose the vector of error terms is distributed with zero covariance among the error terms and a corresponding $n \times 1$ variance vector, σ_t^2. The variance vector is assumed to follow the process:

$$\sigma_t^2 = v + \rho_1 \varepsilon_{t-1}^2 + \dots + \rho_m \varepsilon_{t-m}^2 + u_t , \tag{2}$$

where u_t is an error term vector, each element of which is assumed to be white noise. To test for ARCH effects, we consider an equation in which the equation (2) terms are replaced with their residual counterparts, i.e.

$$\hat{\varepsilon}_t^2 = v + \rho_1 \hat{\varepsilon}_{t-1}^2 + \dots + \rho_m \hat{\varepsilon}_{t-m}^2 + u_t , \tag{3}$$

where $\hat{\varepsilon}_t^2$ is the vector of squared residuals at time t. The null hypothesis of no multivariate ARCH effects of degree m is $\mathrm{H}_0 : \rho_1 = \rho_2 = \dots = \rho_m = 0$ and the alternative hypothesis of multivariate ARCH effects of degree m is at least one of the ρ_i matrixes (i = 1, . . ., m) is not a zero matrix. The following multivariate LaGrange Multiplier (*MLM*) test statistic can be used to test the null hypothesis:

$$MLM(m) = (T - \Delta)\log\left(\frac{|\Omega_R|}{|\Omega_U|}\right) \tag{4}$$

where the denotation Δ is an adjustment for parameters,[1] and T is the sample size. Ω_R is the estimated variance-covariance matrix for the error term vector in equation (3) when the null hypothesis of no multivariate ARCH(m) is imposed. Ω_U is the estimated variance-covariance matrix for the error term vector in equation (3) when the null hypothesis is not imposed. $|\Omega|$ represents the determinant of matrix Ω. Under the null hypothesis of no multivariate ARCH effects of order m, the LM statistic is asymptotically distributed as χ^2 with $m \times n^2$ degrees of freedom.[2]

3. Monte Carlo Simulation and Bootstrapping

For the purpose of simulations in this study we will concentrate on a bivariate VAR(1) model with multivariate ARCH effects of first degree as the following:

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix} + \begin{bmatrix} \alpha_{1,11} & \alpha_{1,12} \\ \alpha_{1,21} & \alpha_{1,22} \end{bmatrix} \times \begin{bmatrix} y_{1t-1} \\ y_{2t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}, \tag{5}$$

where each ε_{it} (i = 1, 2) is independently drawn from N(0, σ_{it}^2), and

$$\begin{bmatrix} \sigma_{1t}^2 \\ \sigma_{2t}^2 \end{bmatrix} = \begin{bmatrix} 1-\rho_{1,11} \\ 1-\rho_{1,22} \end{bmatrix} + \begin{bmatrix} \rho_{1,11} & 0 \\ 0 & \rho_{1,22} \end{bmatrix} \times \begin{bmatrix} \varepsilon_{1t-1}^2 \\ \varepsilon_{2t-1}^2 \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}. \qquad (6)^{3}$$

The error terms u_{it} (i = 1, 2) are drawn independently from a standard normal distribution. To allow us to use the same number of observations regardless of number of observations and to remove effects from start up values, we generate 100 presample observations. We use a variety of combinations of parameter values in the "true" model.

Table 1 displays the various values for the parameters that we use in equation (5). Enumerating all the combinations of these parameters, we come up with 294 (6×7×7) combinations.

[1] The adjustment is done by making use of an Edgeworth expansion suggested by Anderson (1958), which is generalized by Hatemi-J (2004). In our case, Δ = T - m×n + 0.5(n(m-1)-1).

[2] Johansen suggested the test statistic in (4) for testing multivariate autocorrelation, and indicates it is asymptotically distributed as χ^2 with $m \times n^2$ degrees of freedom. We use this test statistic for testing another constraint on the multivariate system, so it should have the same asymptotic distribution.

[3] For more details regarding the derivation of the multivariate ARCH effects as described by equation (6) see Hatemi-J (2004).

Table 1. Parameter values for VAR model of equation (4)

$\alpha_{1,11}$	-1	-0.6	-0.2		0.2	0.6	1.0
$\alpha_{1,22}$	-0.8	-0.5	0.1	0.0	0.1	0.5	0.8
$\alpha_{1,12} = \alpha_{1,21}$	-0.8	-0.5	0.1	0.0	0.1	0.5	0.8

We run simulations separately for two assumptions on the ARCH effects. One, to check the size properties, is to assume the true model has no ARCH effects, i.e., $\rho_{1,11}= \rho_{1,22}=0$. The other, to consider power properties, is to assume the true model has first-degree ARCH effects with $\rho_{1,11}= \rho_{1,22}=0.5$. In general, having the intercept vector in the multivariate ARCH process as $(1-\rho_{1,11}, 1-\rho_{1,22})'$, with the ARCH coefficient terms between 0 and 1 inclusive as we have, guarantees the unconditional variance for each error term is equal to one.

We also run our simulations separately for those parameters values resulting in stability of the model and those that do not. The distinction between stable and unstable cases is relevant, as stability of the model is important for standard asymptotic theory to apply, but instability is common in economic time series. By looking at the modulus of the companion matrix for

$$B = \begin{bmatrix} \alpha_{1,11} & \alpha_{1,12} \\ \alpha_{1,21} & \alpha_{1,22} \end{bmatrix}, \tag{7}$$

we can determine the stability of the model (the modulus is the square root of the summed squares of the real and imaginary eigenvalue elements). Stability is indicated when the modulus of each eigenvalue is less than unity. Otherwise instability is indicated. Parameter value combinations that result in explosive processes, with a maximum absolute-value modulus greater than 1, are dropped. All but 150 of the parameter-value combinations are dropped for this reason. Table 2 indicates the distribution of the remaining parameter cases for various levels of the maximum absolute-value modulus.

Table 2. Distribution of parameter cases with respect to the modulus

STABILITY STATUS	NUMBER OF CASES	PERCENT OF CASES WITH MAXIMUM ABSOLUTE-VALUE MODULUS FALLING WITHIN RANGE.					
		<0.25	[0.25,0.5)	[0.5, 0.75)	[0.75,0.95)	[0.95,1)	1
Stable	136	10.29	2.94	39.71	41.18	5.88	0.00
Unstable	14	0.00	0.00	0.00	0.00	0.00	100

In addition to distinguishing between stable and unstable models and between models with and without multivariate ARCH effects our simulations, we also distinguish between two sample sizes, T, of 40 and 100. This leads to eight scenarios consisting of the eight combinations of the two different stability statuses, the two different sample sizes, and the two different assumptions about ARCH effects.

For each case in each of the eight scenarios we perform 1000 simulations. In each of these simulations we estimate each of the two scalar equations implied in (5) using OLS, resulting in residuals for each equation $\hat{\varepsilon}_{it}$ (i = 1, 2 and t = 1, . . . , T) and the estimated squared residual vector $\hat{\varepsilon}_t^2 = (\hat{\varepsilon}_{1t}^2, \hat{\varepsilon}_{2t}^2)'$. With the estimated squared residual vector we estimate (3) with and without the restriction $\mathrm{H}_0 : \rho_1 = \rho_2 = \Lambda = \rho_m = 0$ using OLS, find the estimated variance-covariance matrix under each situation, and compute the *MLM* test statistic as shown in equation (4). In each of these simulations we test for ARCH effects by determining whether the *MLM* statistic is significantly large according to the χ^2 as described earlier and according to quantiles from a bootstrap distribution to be described below. The size and power properties of these tests are investigated at the nominal levels of 1%, 5%, and 10% for m = 1, . . ., 5.

The procedure that we use to determine the bootstrap distribution consists of the following steps:

1. For each scalar equation implied in (5), T values are drawn randomly and independently with replacement from the residuals for that equation, $\hat{\varepsilon}_{it}$, and the mean of those selected values is subtracted from each value to give the bootstrap residuals, ε_{it}^*.
2. The bootstrap values for y_t, denoted by y_t^*, are computed by using the equation
   $$y_t^* = \hat{c} + \hat{A}_1 y_{t-1} + \varepsilon_t^*,$$
   where a circumflexed parameter represents the OLS estimate for that parameter and $\varepsilon_t^* = (\varepsilon_{1t}^*, \varepsilon_{2t}^*)'$.
3. The bootstrap *MLM* (*BMLM*) is calculated using the y_t^* series (t = 1, . . ., T), calculating the statistic the same way as we do using the "true" simulated data.
4. Steps 1-3 are repeated 500 times to generate the bootstrap distribution of the test statistic.

4. The Results of the Monte Carlo Experiments

The simulation results for size properties of *MLM* test and the *BMLM* test are presented in Tables 3-6 and the results for power properties of these two tests are presented in Tables 7-8. In this study two measures are used to check the size properties of the tests. The first measure is the estimated actual size. The second measure is absolute deviations from the nominal size. The means over the parameter combinations in the particular scenario are presented for both measures. Based on the simulation results the following can be noted.

Using the asymptotic distribution, the actual size tends to be higher than the nominal size when the nominal size is 1%, but the reverse is generally true when the nominal size is 5% or 10% and the sample size is small (40 observations) and/or the VAR model is stable. Also, when the sample size is small and/or the VAR model is stable, and the nominal size is 5% or 10%, using the bootstrap distribution rather than the asymptotic one generally results in the actual size being closer to the nominal size. Similar conclusions can be drawn when one looks at the average absolute deviation from the nominal size for the asymptotic and bootstrap multivariate *LM* tests. It is of course a subjective issue, but the average absolute deviations do not seem very excessive for the asymptotic *LM* tests. For the moderate sample size ($T = 100$) with unstable VAR models for example, they are 0.4 to 0.7 percentage points at the 5% nominal size, and 0.8 to 1.6 percentage points at the 10% nominal size. However, they are 0.4 to 0.7 percentage points at the 1% nominal size in this example, which seems somewhat high at this level.

At the 5% and 10% nominal size level, the power of the bootstrapped *LM* test is superior to the asymptotic one when the sample size is small and/or the multivariate ARCH lag order of the test is equivalent to that of the true model (we test for power only in the situation where the true model has a 1-lag ARCH structure, however). At the 1% nominal size level, the power of the asymptotic *MLM* test is superior of the bootstrap version, and this is likely due to the actual size tending to be higher than the actual size when using the asymptotic test at this nominal size level. Another result that can be drawn from the simulations is that, unsurprisingly, larger sample sizes produce higher power irrespective of whether the bootstrap distribution or the asymptotic distribution is used. Also in unstable VAR models the power seems to be higher compared to stable VAR models for the same sample size.

5. Conclusions

This study has extended Engle's *LM* test for ARCH effects to a multivariate test. The size and power properties of this test are investigated for testing multivariate ARCH effects in the stable and unstable VAR models. We have shown that for the typical nominal size used in business and economics of 5% that the bootstrapped version of the multivariate *LM* test has relatively better size and power properties in small samples and/or stable VAR models. However the gains from using the bootstrap version of the multivariate *LM* test are not clear, especially when using 1% nominal size, where bootstrapping leads to little if any gain in achieving the appropriate size and leads to slightly worse power performance, and when dealing with unstable VAR cases with moderate sample sizes.

To our best knowledge any test for multivariate ARCH effects is not available automatically in the common econometric packages in the market, unlike other multivariate diagnostic tests, e.g. tests for autocorrelation and normality. Given what we see in our simulation results, we think that making at least the asymptotic multivariate *LM* test available in software packages would be useful for practitioners. We find this important because OLS estimation for VAR, which is the standard VAR estimation method, results in estimates that are not consistent if ARCH effects are present. Therefore, it its important to test for multivariate ARCH effects when estimating a VAR model.

Table 3: Actual sizes and average absolute deviations from the nominal size when $T = 40$ in stable VAR models

	ACTUAL SIZES FOR EACH TEST					ABSOLUTE DEVIATION FROM NOMINAL SIZE				
ARCH order→	1	2	3	4	5	1	2	3	4	5
	Nominal size 1%					Nominal size 1%				
MLM	**1.1%**	1.2%	1.2%	1.3%	1.2%	**0.3%**	0.3%	0.3%	0.3%	0.3%
BMLM	1.3%	1.2%	1.2%	**1.2%**	1.2%	0.4%	0.3%	0.3%	0.3%	0.3%
	Nominal size 5%					Nominal size 5%				
MLM	3.6%	4.2%	4.3%	4.4%	4.4%	1.4%	0.9%	0.8%	0.7%	0.7%
BMLM	**5.0%**	**5.1%**	**5.2%**	**5.2%**	**5.3%**	**0.7%**	**0.5%**	**0.6%**	**0.6%**	**0.6%**
	Nominal size 10%					Nominal size 10%				
MLM	6.7%	7.7%	7.7%	8.0%	8.0%	3.3%	2.3%	2.3%	2.0%	2.0%
BMLM	**9.5%**	**9.7%**	**9.9%**	**10.1%**	**10.2%**	**1.0%**	**0.8%**	**0.7%**	**0.7%**	**0.8%**

MLM is the multivariate LaGrange Multiplier presented in equation (3) and *BMLM* is the bootstrapped *MLM*. Bold values indicate best relative performance. The means over the parameter combinations in the particular scenario are presented for both measures.

Table 4: Actual sizes and average absolute deviations from the nominal size when $T = 40$ in unstable VAR models

	ACTUAL SIZES FOR EACH TEST					ABSOLUTE DEVIATION FROM NOMINAL SIZE				
ARCH order→	1	2	3	4	5	1	2	3	4	5
	Nominal size 1%					Nominal size 1%				
MLM	1.3%	1.4%	1.4%	1.4%	1.4%	0.4%	0.4%	0.4%	0.4%	0.4%
BMLM	1.3%	**1.2%**	**1.2%**	**1.2%**	**1.3%**	**0.3%**	**0.3%**	**0.3%**	**0.3%**	0.4%
	Nominal size 5%					Nominal size 5%				
MLM	4.2%	4.7%	4.6%	4.6%	4.5%	0.9%	**0.4%**	0.5%	0.7%	0.7%
BMLM	**5.1%**	**5.3%**	**5.1%**	**5.1%**	**5.2%**	**0.7%**	0.5%	**0.5%**	**0.5%**	**0.6%**
	Nominal size 10%					Nominal size 10%				
MLM	7.4%	8.2%	8.3%	8.3%	8.4%	2.6%	1.8%	1.8%	1.7%	1.6%
BMLM	**9.7%**	**10.0%**	**10.2%**	**9.9%**	**10.3%**	**1.0%**	**0.6%**	**0.9%**	**0.9%**	**0.9%**

MLM is the multivariate LaGrange Multiplier presented in equation (3) and *BMLM* is the bootstrapped *MLM*. Bold values indicate best relative performance. The means over the parameter combinations in the particular scenario are presented for both measures.

Table 5: Actual sizes and average absolute deviations from the nominal size when $T = 100$ in stable VAR models

	ACTUAL SIZES FOR EACH TEST					ABSOLUTE DEVIATION FROM NOMINAL SIZE				
ARCH order→	1	2	3	4	5	1	2	3	4	5
	Nominal size 1%					Nominal size 1%				
MLM	1.3%	1.4%	1.4%	1.4%	1.3%	**0.3%**	0.5%	0.4%	0.5%	0.4%
BMLM	1.3%	**1.3%**	**1.3%**	**1.3%**	1.3%	0.4%	**0.4%**	0.4%	**0.4%**	0.4%
	Nominal size 5%					Nominal size 5%				
MLM	4.3%	4.7%	4.8%	4.8%	4.6%	0.8%	0.6%	0.6%	**0.5%**	0.6%
BMLM	**5.2%**	**5.2%**	**5.3%**	5.2%	**5.3%**	**0.5%**	0.6%	0.6%	0.6%	0.6%
	Nominal size 10%					Nominal size 10%				
MLM	7.9%	8.6%	8.6%	8.7%	8.5%	2.1%	1.5%	1.4%	1.3%	1.5%
BMLM	**9.9%**	**10.1%**	**10.1 %**	**10.1%**	**10.1 %**	**0.7%**	**0.8%**	**0.7%**	**0.7%**	**0.8%**

MLM is the multivariate LaGrange Multiplier presented in equation (3) and *BMLM* is the bootstrapped *MLM*. Bold values indicate best relative performance. The means over the parameter combinations in the particular scenario are presented for both measures.

Table 6: Actual sizes and average absolute deviations from the nominal size when $T = 100$ in unstable VAR models

	ACTUAL SIZES FOR EACH TEST					ABSOLUTE DEVIATION FROM NOMINAL SIZE				
ARCH order→	1	2	3	4	5	1	2	3	4	5
	Nominal size 1%					Nominal size 1%				
MLM	1.5%	1.6%	1.6%	1.7%	1.4%	0.5%	0.6%	0.6%	0.7%	0.4%
BMLM	**1.4%**	**1.4%**	**1.4%**	**1.4%**	**1.3%**	**0.4%**	**0.4%**	**0.4%**	**0.4%**	0.4%
	Nominal size 5%					Nominal size 5%				
MLM	**4.8%**	**5.3%**	**5.4%**	**5.3%**	**4.8%**	**0.6%**	**0.4%**	**0.7%**	**0.6%**	0.6%
BMLM	5.4%	5.6%	5.6%	5.6%	5.4%	0.7%	0.7%	0.8%	0.8%	0.6%
	Nominal size 10%					Nominal size 10%				
MLM	8.4%	9.3%	**9.5%**	9.2%	8.7%	1.6%	0.9%	0.8%	1.0%	1.3%
BMLM	**10.1%**	**10.6 %**	10.7%	**10.5 %**	**10.2 %**	**0.9%**	**0.8%**	0.8%	**0.6%**	**0.7%**

MLM is the multivariate LaGrange Multiplier presented in equation (3) and *BMLM* is the bootstrapped *MLM*. Bold values indicate best relative performance. The means over the parameter combinations in the particular scenario are presented for both measures.

Table 7: Power properties when $T = 40$

	STABLE VAR MODEL					UNSTABLE VAR MODEL				
ARCH order→	1	2	3	4	5	1	2	3	4	5
	Nominal size 1%					Nominal size 1%				
MLM	**17.4%**	**14.0%**	**11.5%**	**10.0%**	**8.8%**	**19.6%**	**16.1%**	**13.1%**	**11.8%**	**10.2%**
BMLM	16.7%	12.3%	10.0%	8.5%	7.5%	17.0%	12.8%	10.5%	9.2%	8.2%
	Nominal size 5%					Nominal size 5%				
MLM	30.0%	25.3%	21.3%	19.0%	17.1%	32.9%	27.6%	23.6%	21.4%	19.1%
BMLM	**34.0%**	**26.9%**	**22.7%**	**20.0%**	**18.1%**	**35.0%**	**28.1%**	**24.1%**	21.4%	**19.4%**
	Nominal size 10%					Nominal size 10%				
MLM	38.4%	32.9%	28.5%	25.9%	23.5%	41.4%	35.6%	31.2%	28.4%	25.8%
BMLM	**44.6%**	**36.9%**	**32.1%**	**28.9%**	**26.6%**	**46.4%**	**38.5%**	**34.0%**	**30.7%**	**28.2%**

MLM is the multivariate LaGrange Multiplier presented in equation (3) and *BMLM* is the bootstrapped *MLM*. Bold values indicate best relative performance. The means over the parameter combinations in the particular scenario are presented for both measures.

Table 8: Power properties when $T = 100$

	STABLE VAR MODEL					UNSTABLE VAR MODEL				
ARCH order→	1	2	3	4	5	1	2	3	4	5
	Nominal size 1%					Nominal size 1%				
MLM	**71.7%**	**63.8%**	**57.3%**	**52.2%**	**47.5%**	**73.1%**	**65.2%**	**58.8%**	**53.6%**	**48.8%**
BMLM	66.6%	56.3%	49.6%	44.4%	40.3%	66.1%	56.5%	49.8%	44.4%	40.4%
	Nominal size 5%					Nominal size 5%				
MLM	83.1%	**76.5%**	**70.8%**	**66.1%**	**61.7%**	84.2%	**77.6%**	**72.1%**	**67.4%**	**63.0%**
BMLM	**84.1%**	75.8%	69.6%	64.5%	60.4%	**84.3%**	76.2%	70.0%	65.1%	61.3%
	Nominal size 10%					Nominal size 10%				
MLM	88.0%	82.4%	77.4%	73.1%	69.3%	88.9%	83.4%	78.7%	74.2%	70.5%
BMLM	**89.9%**	**83.6%**	**78.6%**	**74.1%**	**70.4%**	**90.2%**	**84.3%**	**79.2%**	**74.6%**	**71.0%**

MLM is the multivariate LaGrange Multiplier presented in equation (3) and *BMLM* is the bootstrapped *MLM*. Bold values indicate best relative performance. The means over the parameter combinations in the particular scenario are presented for both measures.

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